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Report on the Application of Finite Element Analysis to Historic Structures

Westminster Hall, London

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Monumental architecture and the design conceptions on which it is based inevitably have structure at their very core. The spacing of columns, dimensions of beams, thickness of walls, and placement of buttresses—the elements which define our spatial experience—all require consideration of structural parameters. As a result, the form and the structure of many great monumental works of architecture are tightly interwoven, so integrated that any attempt to study form or style in such buildings must be based on an understanding of the structural principles at work.

Modern historians exploring earlier building technology have several options regarding structure, its relationship to aesthetic developments, and its place in architectural history. The first is simply to ignore it; a common enough choice with the development of art and architectural history as a separate discipline as part of the liberal arts curriculum. For many historians, however, this is not an entirely satisfactory solution. Historic builders and designers striving to ensure the integrity of their constructions had no such luxury; their works inevitably grappled with both structural and aesthetic issues at once.

Alternatively, the researcher may choose to engage this relationship seriously, relying on his or her knowledge of the period's construction practices and techniques to interpret the significance of structural decisions in historic buildings. This, perhaps, has been the most common approach. For example, inquiry into the logic of a given structural configuration or device may provide a rudimentary understanding of a building's structural performance. With simple structures—and in this category fall most medieval timber roofs—this practice may be enough to highlight the relationships between design decisions and structural considerations. However, the accurate determination of a more complex building's structural behavior typically requires the technical knowledge of those trained in engineering. Hence, in the field of architectural history, this path has been pursued by only a few.[1]

Historians may also collaborate with engineers. Unfortunately, the engineering process (along with its specialized terminology and numeric answers to technical questions) has often discouraged nonengineers from active participation in analysis. Moreover, prior to the development of computer-aided analysis in the 1960s, engineering had some serious limitations for historical research: Lengthy hand calculations characterized the process, and to facilitate computation theoretical structural models were often developed on the basis of oversimplified assumptions.[2] Finally, as alternative hypotheses could not quickly be explored for their validity, they were not generally explored at all.

This paper describes the potential of another approach to structural insight, one in which computer-aided finite element analysis (FEA) is used to inform historical research directly. FEA is an interactive medium effective in exploring and discussing structural hypotheses. With minimal training, it is accessible to nonengineers. Its products include output useful to developing an understanding of a structure's overall behavior and details concerning the performance of its components. Most important, by enabling researchers trained in architectural history to participate directly in the structural analysis of historic buildings, it can highlight the relationships between formal or aesthetic issues and structural developments.

WESTMINSTER HALL

As an illustration of this method the authors used FEA to explore the trusses at Westminster Hall. Hugh Herland's celebrated roof (1395–96) is best known for uniting two hitherto separate roof prototypes—the hammerbeam and the arch-brace-and-collar roof—to create a structure spanning one-and-a-half times the distance of any previous timber roof [*Figure 1*]. It is also famous for its highly decorative traceried woodwork. How does the truss carry its loads and with what intent were its ornamental features employed? This structure of innovative design and outstanding beauty has been the subject of a great deal of speculation by historians, engineers, and architects alike.[3]

A century of numerous studies, analyses, and debate, however, has only begun to establish the overall load–bearing behavior of Herland's complex design. Beginning in 1914 with an exhaustive study by Sir Frank Baines (Baines's *Report*)

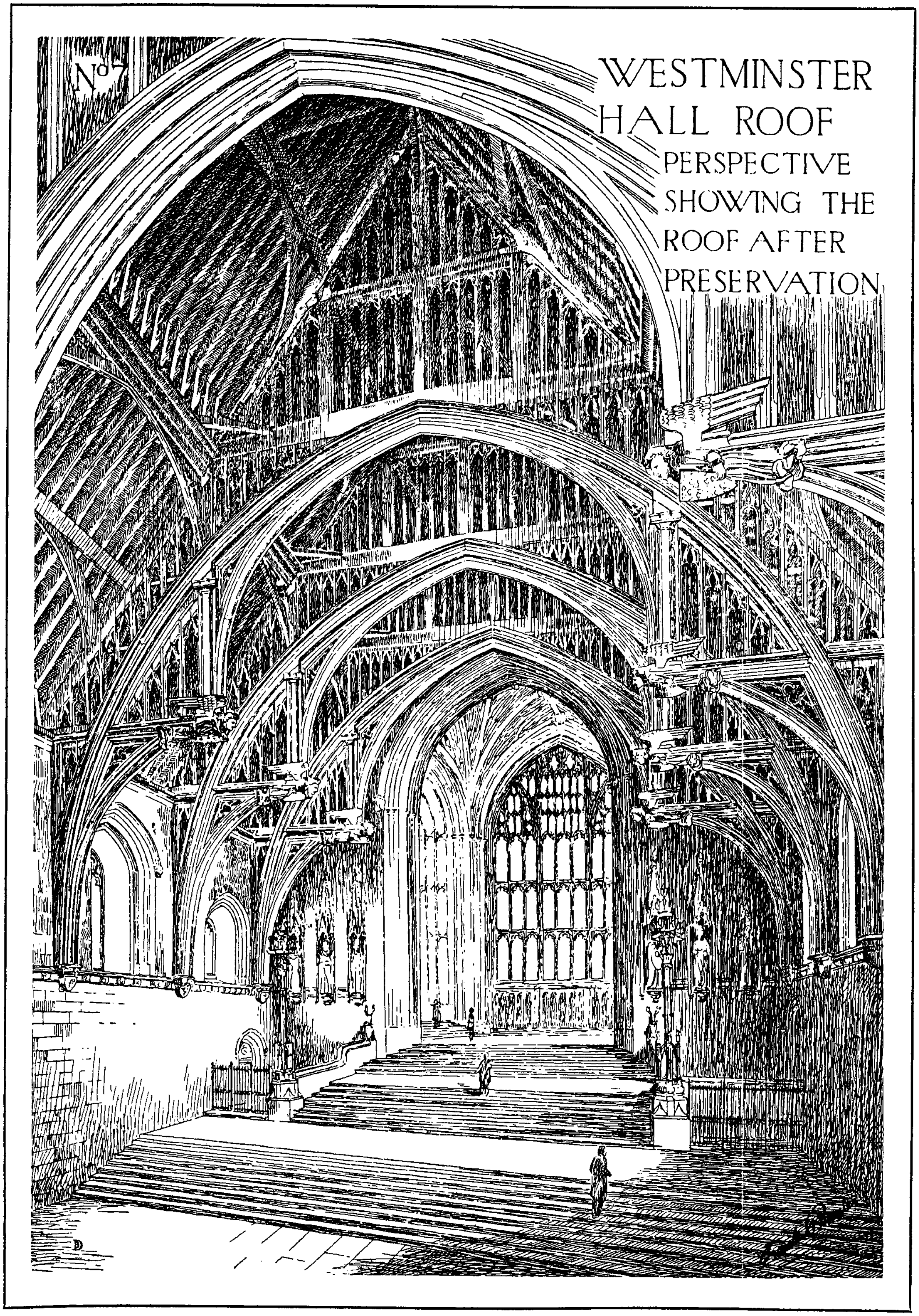

FIGURE 1: Sir Frank Baines, interior perspective, Westminster Hall roof, from *Report on . . . Westminster Hall* (1914).

documenting the condition of the roof trusses, twentieth-century investigators have attempted to explain the performance of the truss.[4] As architect in charge of roof restorations for Westminster Hall (1913–22) Baines was primarily interested in establishing the viability of the much decayed trusses and determining the most effective means of reinforcing them. His analysis of the structure was based primarily on his general knowledge of hammerbeam and timber–frame construction.[5] In 1926, A. J. Sutton Pippard and W. H. Glanville used the Westminster roof as a case study to demonstrate Pippard's pioneering "strain energy stress analysis" technique.[6] In 1967, engineers Jacques Heyman and Rowland Mainstone each took up the discussion and questioned which of the principal members in Herland's roof were structural and what constituted appropriate assumptions for analysis.[7] Most recently, in 1987 Robert Mark and his associates, in collaboration with Lynn Courtenay, conducted the first structural study of the trusses to combine both scale model and computer analyses.[8] Although all of these researchers have brought the best of available theory and science to the task of explaining Herland's remarkable technical achievement, there is as yet no definitive analysis.[9]

This study continues this tradition of structural investigation by bringing state-of-the-art engineering analysis to the problem of understanding the trusses' structural behavior. It differs from those preceding it, however, in one important respect: it includes the ornamental tracery in the analysis of possible structural components [*Figure 2*]. Either because of the difficulty of analyzing it with earlier techniques, or because of a bias held by engineers and historians which has viewed the tracery as purely ornamental, no one has studied the truss in its entirety.[10] This is understandable considering the tracery's slight section in comparison to those of the massive principal timbers which characterize the truss. The great arch, for example, composed of three pegged sections of English oak, is nearly two foot square in section, lending a powerful presence to the truss. By contrast, the tracery seen 60 and 80 feet above the floor presents a delicate filigree of detail tying together such principal timbers. Each individual puncheon of the tracery, however, is a sizable timber in its own right, and at roughly 4 by 9 inches in section is capable of sustaining thousands of pounds of compressive force.

The Finite Element Method of Structural Analysis

The creation of the analytical technique now known today as the finite element method (FEM) spanned a century to combine the computational power of the digital computer with developments in continuum mechanics.[11] FEM breaks down any structural system or continuum into submembers (such as individual truss members or discrete segments of a continuous vault). Computer algorithms then utilize the spatial information locating these discrete elements and the mechanical properties of their materials to assemble a comprehensive model so that an overall structural evaluation can be made (see Appendix: On the finite element method of analysis).

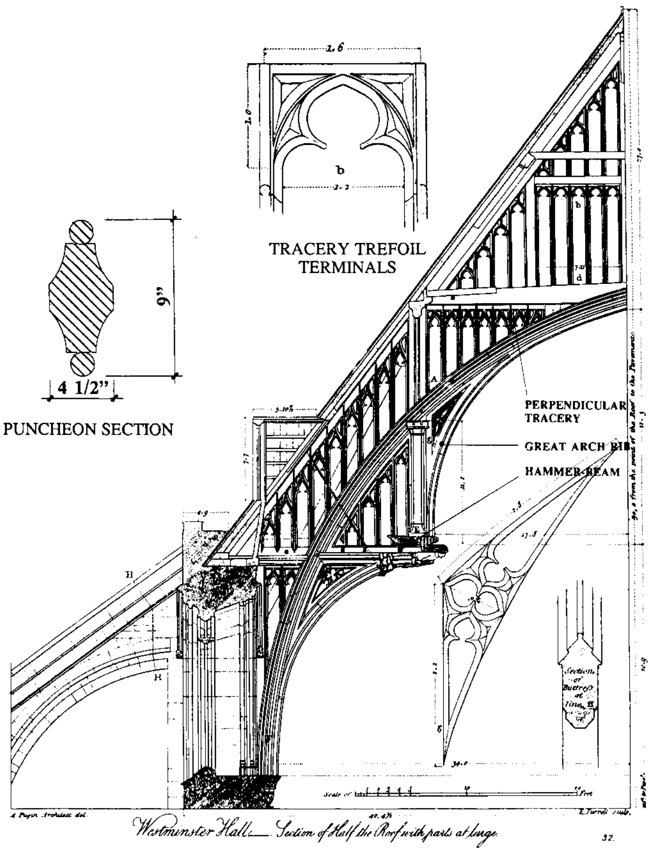

FIGURE 2: A. C. Pugin, truss and tracery details, Westminster Hall roof, from *Specimens of Gothic Architecture* (1895); section of tracery puncheon added by authors.

In the field of structural engineering, finite element analysis has rapidly become the standard: it has also been used to study such diverse phenomena as the structure of marine shelled organisms, stresses in bio-engineered human in-plants, the air flow around airplane wings and the forces in a suspension bridge.[12] FEA supersedes many of the former methods historians may be familiar with (including plastic modeling).[13] It is far more precise, cheaper and provides quicker results. It is also more flexible: FEA can be used to investigate entire buildings constructed of any material of known properties (stone, concrete, unit masonry, steel, iron, or wood) or explore the performance of a building's subassemblies (domes, buttresses, or trusses). Finally, because of FEA's computational power, the number of simplifying assumptions necessary for analysis is greatly reduced, leading to more accurate results. These strengths, which have caused FEA to revolutionize engineering practice, are equally beneficial to historic structures research, and as a result FEA is beginning to be used in historical applications.[14]

Perhaps the strongest point for historians without formal training in engineering is that FEA output is graphic and intuitive. Once the structural model is coded and run on a computer the user can essentially view the internal structural workings of the building firsthand, including diagrams of its deflected shape under loading, the axial forces (running parallel with its members), bending moments, and shear forces (running perpendicular to the axes of members). A series of "what if" games can then be played with the working computer model. Changes in support or loading assumptions are easily made and the results quickly obtained. For example, the numerous alterations made to a Gothic cathedral over the centuries can be explored (do they reflect formal or structural concerns?). Herein lies the potential of FEA to transform our understanding of a historic building from one which is focused solely on formal, stylistic, or contextual issues to one in which the real structural problems addressed by the builders and the designers of the time are given their appropriate weight.

Building the Finite Element Model

Our method of study was to build a finite element model of the Westminster Hall truss in order to explore its performance and the possible structural contribution of its tracery. This process was interactive and included

- building the basic finite element model and using it to help establish informed assumptions about its construction, loading, and support
- analyzing the structure with and without tracery to isolate its structural contribution
- interpreting the historical significance of our findings.

The basic FEA model was built in two–dimensional Cartesian space (*x* and *y* coordinates), in this case defining the geometry of the truss and the location of joining timbers at "nodes." The truss members were represented as model "frame elements," given their specific dimensions and material properties in the original, undecayed truss, and attached to these nodes. With this model in place we were able to explore the behavior of the truss with and without tracery and under varied loading and support assumptions.

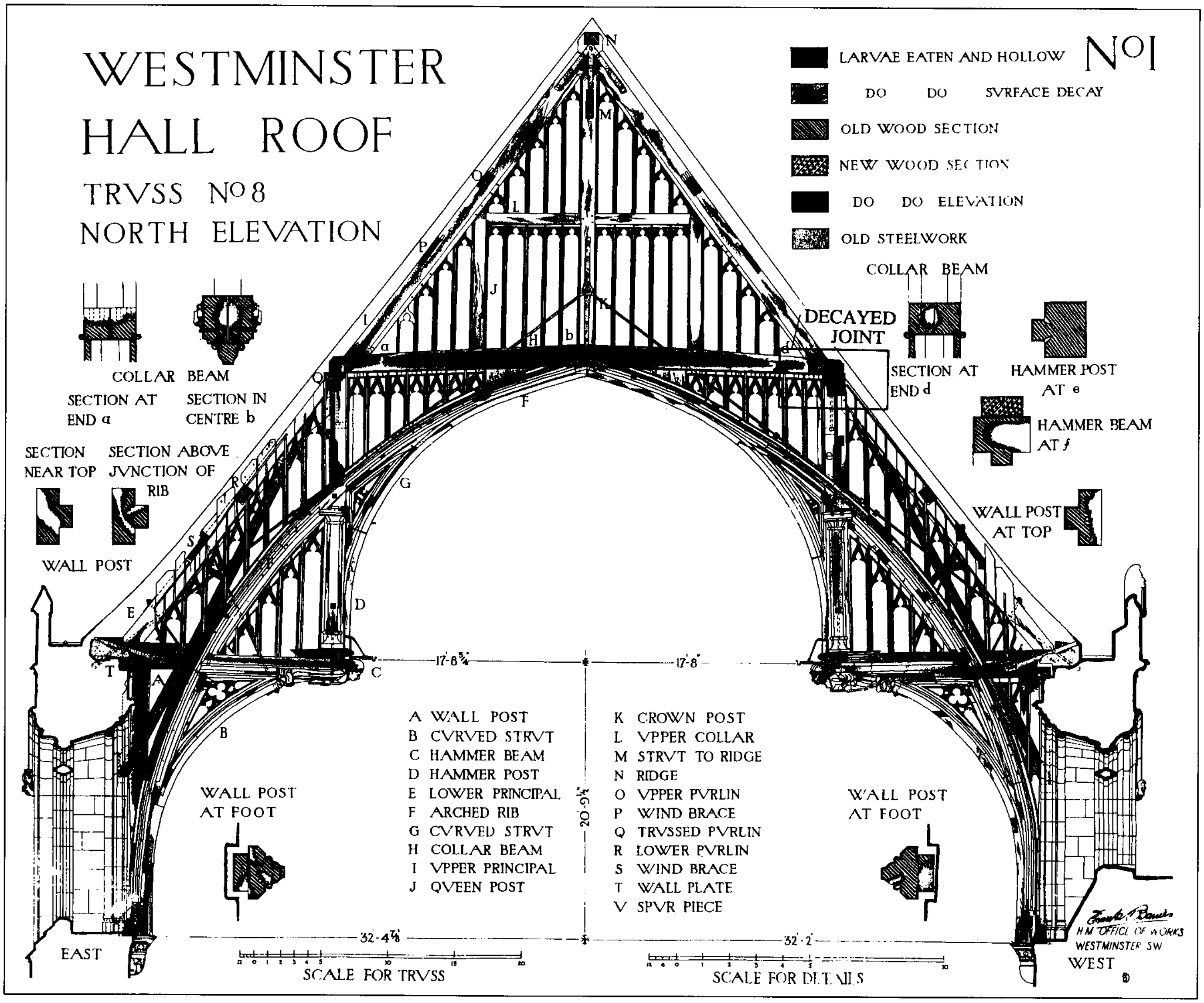

FIGURE 3: Sir Frank Baines, truss elevation and detail of collar beam, Westminster Hall roof, from *Report on . . . Westminster Hall* (1914).

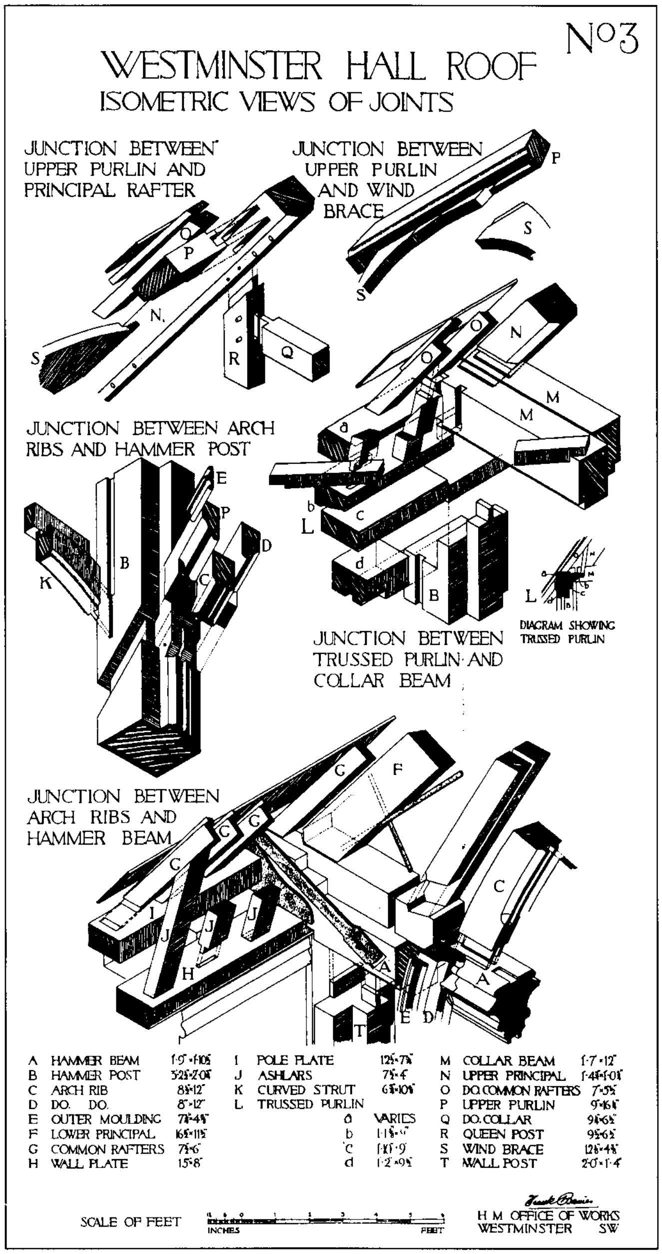

FIGURE 4: Sir Frank Baines, isometric views of joints, Westminster Hall roof, from *Report on . . . Westminster Hall* (1914).

Baines's *Report* and his recently recovered "schedules" (drawings executed in preparation for his restoration of the roof) formed the basis of our model of Herland's 1395–96 truss [*Figures 3–5*].[15] These resources were supplemented by drawings of the truss and its members in works by A. C. Pugin and E. J. Willson, Friedrich Ostendorf, H. Cescinsky, and E. R. Gribble.[16] The large truss members were typically mortised, tenoned, and pegged together, forming stiff joints capable of resisting substantial forces. These connections were modeled as fixed, released, constrained, or partially released, according to the best mathematical representation for the physical condition. For example, the composite great arch is pegged together at many points. This member was modeled as three separate truss-frame members constrained to move together at those pegged points. The connection where the outer laminae of the great arch pass through mortises in the hammer post was modeled as a rigid moment connection, one capable of transmitting rotational forces to all adjoining elements (see Figure 4, detail entitled "Junction between Arch Ribs and Hammer Beam"). The remaining joints, with the exception of the truss connection to the masonry walls, were modeled with three degrees of freedom (free to rotate about the orthogonal z axis and translate in the x–y plane of the truss).

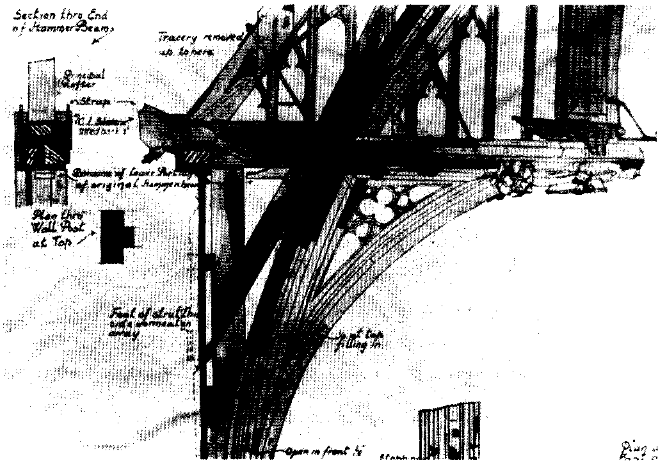

FIGURE 5: Sir Frank Baines, section and elevation of truss at wall head, Westminster Hall roof. Schedule no. 27 made in preparation for truss reinforcements. Note the air gap between the masonry wall and wall post with blocking providing lateral support at its midpoint.

The usefulness of any structural model, with the finite element model no exception, is dependent on accurate assumptions concerning its support and loading. Engineering models are abstractions of the structures they represent; and while the determination of precise stress levels in members may not be the main issue in the analysis of a historic building, the basic assumptions must be grounded in the reality of the structure. But what is this reality?

First is the issue of support. A lively debate between engineers over realistic support assumptions for the truss has effectively been put to rest by the recovery in 1990 of Baines's schedules.[17] During the 1395–96 building campaign the Norman walls were raised and heraldic corbel stones installed approximately 20 feet below the new wall head, providing seating at the base of the truss's wall posts. While each truss appears to bear both on these corbel stones and at the wall head, many support alternatives can be and have been argued.[18] The contested issue among investigators has been where the walls resist the outward thrust of the truss. Baines's schedules provide strong evidence that (contrary to Heyman's hypotheses) no lateral bearing can occur where the truss apparently abuts the inside corner of the wall head. Schedule 27 [*Figure 5*] shows that the upper half of the wall post was set several inches away from the masonry wall and that as a result the notch between the hammerbeam and wall post could never make contact with the wall head.[19] Further, lateral support of the wall post is shown to be provided at its base and by blocking behind its midpoint, at the approximate location of the springing of the great arch. But what of vertical load bearing? In the absence of lateral support at the wall head our computer models indicated little difference in the structural performance of the truss whether or not the hammerbeam is assumed to bear the truss and roof weight on the top of the wall. Hence, we modeled the frame of the truss with support in only two locations: horizontal and vertical support at the wall post's base (the corbel stone) and horizontal support alone at its approximate midpoint.[20] A thorough and invasive site investigation of these conditions or more convincing archival evidence could have resolved the issue of support. Lacking these, FEA was used to test the feasibility of various hypotheses. In this sense it can complement traditional forms of historical evidence.

Early investigators' assumptions about loading were varied and largely arbitrary. In order to simplify his analysis Pippard adopted a pattern of eight point loads for "approximate loadings" [*Figure 6*]. Heyman and Mainstone progressively abstracted Pippard's loading, and Mark, seeking "general behavior," assumed three equal point loads of 10 metric tons applied at the attachment points of the ridge member and the main purlins.[21] As illustrated by Viollet-le-Duc in his *Dictionnaire raisonné de l'architecture française* [*Figure* 7], the roof framing consists of many different–sized purlins braced in various fashions, so that overall, they carry more or less of the roof load and apply forces to the truss according to their stiffness (arrows in figure 7 indicate loading points on the truss). This weight distribution does not result in any regular pattern, as suggested previously. Recognizing that structures are sensitive to loading we performed a separate three-dimensional finite element analysis of the roof to determine a more accurate load pattern with which to model the truss [*Figure 8*].[22] The results of our roof modeling yielded a more complex pattern of self-weight and point loads on the truss [*Figure 9*] than that assumed by previous authors. The trussed purlins were found to carry 72 percent of the roof's vertical loads to the hammer post and hammerbeam. The ridge and the upper and lower purlins carry relatively minor forces to their attachment points on the truss.

RESULTS

Using the loading pattern and support conditions described above we conducted several computer simulations, with and without the presence of tracery. Our studies reveal several characteristics of the behavior of the Westminster Hall trusses and the function of the tracery.

First, the deflected shape diagram indicates what would be expected: the truss, regardless of the presence or absence of tracery, sags under loading from the roof and purlins, exerting an outward thrust on the walls [*Figure 10*]. The FEA graphic output exaggerates the deflected shape to emphasize the direction of movement in the truss and the bowing of its members. These images of bending deformation are the contemporary equivalent of the physical scale models Herland probably used to test the viability of his designs.[23] The only visual description of the flow of forces he could have seen when he pushed on or hung weights from these models would have been those leading to the same types of deflections shown in the FEA image.

Modern engineers associate trusses and truss behavior with axial forces only. When the Westminster Hall truss is examined purely from the point of axial force flow the tracery exhibits little structural contribution [*Figure 11*]. The relative magnitude of forces running parallel with the truss members is indicated in the computer image by thicker bands in those areas of greatest

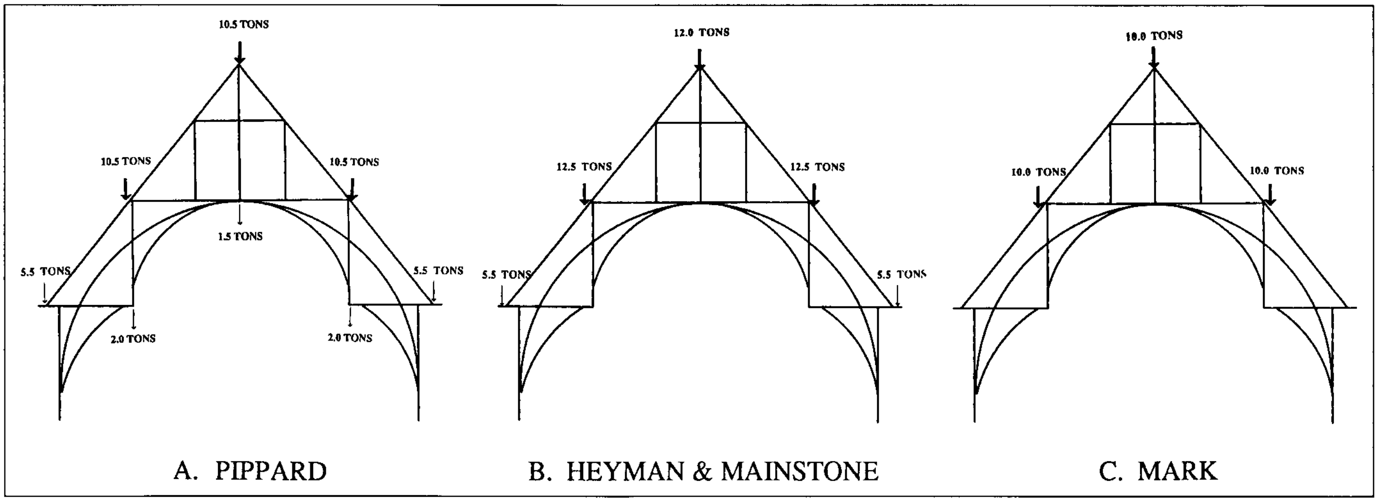

FIGURE 6: Westminster Hall roof, truss loading patterns proposed by previous investigators: (*A*) Eight–point loading, after A. J. Sutton Pippard and W. H. Glanville (1926); (*B*) Jacques Heyman and Rowland Mainstone (1967) and Rowland Mainstone (1967); (*C*) Robert Mark and Huang (1987).